4-5 Determinants

The determinant of a matrix is a value of a square matrix that will be used to:

- calculate the inverse of a matrix
- solve systems

2x2 Matrix (2nd order determinants)
The determinant, D, is calculated as follows:

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

ex:

$$\begin{vmatrix} 5 & 10 \\ 8 & -3 \end{vmatrix} = -15 - 80 = -95$$

ex:

$$\begin{vmatrix} -2 & 6 \\ -5 & 1 \end{vmatrix} = 28$$

$$-2 - -30$$

ex:

$$\begin{vmatrix} 4 & 10 \\ 2 & 5 \end{vmatrix} = 0$$

20

20

ex:

$$\begin{vmatrix} 4 & 6 \\ x & 12 \end{vmatrix} = 12$$

$$48 - 6x = 12$$

$$-6x = -36$$

$$x = 6$$

3x3 Matrix (3rd order determinants)
Befor calculating the determinant, D, we must define a minor.

Minor--of an element in a determinant is the determinant resulting from the deletion of the row and column containing the element.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix}$$

The minor of 5 is: $\begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix}$

The minor of 6 is: $\begin{vmatrix} 5 & -2 \\ 2 & 4 \end{vmatrix}$

The minor of -7 is: $\begin{vmatrix} 5 & -1 \\ 2 & -3 \end{vmatrix}$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix} = 83$$

$$+5\begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix} - -1\begin{vmatrix} 3 & -7 \\ 2 & 4 \end{vmatrix} + -2\begin{vmatrix} 3 & 6 \\ 2 & -3 \end{vmatrix}$$

+21, 24, 3; -14, 12; 12, -9

$$15 + +26 + 42 = 83$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 2 & -1 & -6 \\ 3 & 4 & 2 \\ 5 & -2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} + & - & + \\ & + & \\ & - & \end{vmatrix}$$

$$- -1\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} + 4\begin{vmatrix} 2 & -6 \\ 5 & 3 \end{vmatrix} - -2\begin{vmatrix} 2 & -6 \\ 3 & 2 \end{vmatrix}$$

10, -1, 9; -30, 36, 6; -18, 22, 4

$$-1 \quad + 144 \quad + 44$$

$$= 187$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & 2 & 34 \\ -1 & 3 & 22 \\ 0 & 0 & 4 \end{vmatrix}$$

$\cancel{0|\ |}$ $\cancel{0|\ |}$ $+4\begin{vmatrix} 5 & 2 \\ -1 & 3 \end{vmatrix}$

$= 68$

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15-19,27-31,39

Expansion by minors works for any higher order determinant as well.

ex:

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{vmatrix} = 3$$

$+1\begin{vmatrix} 2 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 0 \\ 3 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}$

$2\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} - 1\left(1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}\right)$

$2 \quad -4 \qquad 1 \quad -6$

$-2 \qquad +5$